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Physics and Logic: Thales

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Thales's protophysics is reconstructed with the help of the Steiner quasigroup. A three-valued logic is proposed, equivalent to it. The discovered connection between physics and logic is used to discuss the natural form of the logic of quantum mechanics.

We need a simple example for tracing the connection of physics with logic. Therefore the earliest physicist, Thales of Miletus (624–537 B.C.), is the focus of our attention. His theory may be put in few words: water is the principle of all. Of course, first of all we must state this theory in a mathematical form. We can come to whatever conclusion only after this.

1. THALES'S PHYSICS

In Thales's physics there is only one physical quantity and the set of its values is $Q' = \{w, b, m\}$. The values are: w, water; b, body; and m, motion. We denote with $x \,\&\, y$ the third, created by the link of x and y.

The basic equations of Thales's physics are

$$x \,\&\, x = x \qquad \text{(the idempotent law)}$$

$$x \,\&\, y = y \,\&\, x \qquad \text{(the commutative law)}$$

$$x \,\&\, (x \,\&\, y) = y \qquad \text{(the Sade keys law)}$$

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The pair $Q = (Q', \&)$ is a well-known (Bruck, 1958) unique Steiner quasigroup with three elements. The operation & has the following Cayley table:

	w	b	m
w	w	m	b
b	m	b	w
m	b	w	m

As an example, we find that $w \& m = b$, or, in words: the third created by the link of water with motion is a body. In short: a body appears of water.

Elements of Q' are theoretical notions, they must be concretized by the explanation of diverse physical facts. Some of these are as follows (Diels, 1951/1952):

1. The fire of the Sun feeds on fumes of the Earth's water. The exact physical description of this fact is $b \& m = w$, where b is the fire of the Sun, m is the feeding, and w is the fumes of the Earth's water.
2. The Earth floats in the World Ocean. The explanation is $b \& w = m$, where b is the Earth, w is the World Ocean, and m is the sailing.
3. All bodies decompose into water. Here $b \& m = w$, where m is the decomposition.

2. PHILOSOPHICAL APPLICATION

Thales's theory is a philosophy also, and we give some philosophical applications of it.

Joannes Grammaticos (1887/1909) was puzzled why Thales never said that the soul arises from water, but Thales's follower Hippo accepted this thesis.

Let x be the soul. We must take into account that the ancients were hylozoists, they considered that all matter is animate. So, the soul satisfies the equation of hylozoism

$$b = x \& m$$

where m is penetration. The debatable thesis, accepted by Hippo and not founded by Thales, is

$$x = w \& m$$

where m is the arising. With the help of Sade's keys law we get $b = (w \ \& \ m) \ \& \ m = w$, which is a contradiction. So, the debatable thesis is contradictory. Seemingly, the astute Thales felt this and did not adduce this thesis.

3. THREE-VALUED LOGIC OF THALES

The physics of Thales may be considered as a logic, too. We dare say that Q' is the set of the constant propositions. Whereas in physics w denotes the chemical element, it denotes the proposition on "this is a water" in logic. At the same time Q' is the set of truth-values. We declare one of the truth-values T as the truth. Of course, $T = w$, $T = b$, or $T = m$. The operation & is now a logical connective, defined by the above Cayley table.

The equality sign is meant in logic as a connective, defined by

$$(x = y) \underset{\text{def}}{=} (x \ \& \ (T \ \& \ y))$$

As an example, the equation of hylozoism must be considered as the proposition $b \ \& \ (T \ \& \ (x \ \& \ m))$ in logic.

We call a proposition x true in Thales's logic if $x = T$. It is easy to see that $x = y$ is correct in Thales's physics if and only if the proposition $x = y$ is true in Thales's logic. Conversely, the truth of a proposition x may be interpreted as a physical equality $x = T$.

We see that Thales's physics and logic are equivalent theories for any choice of T.

4. QUANTUM LOGIC?

We have seen that the reduction of a physical theory into some logic is in the realizing of physical notions as logical ones. What logic arises from elementary quantum mechanics (EQM)?

Let EQM be the quantum theory of a physical system with a sole degree of freedom. We consider the countable set $\Psi = \{\psi_0, \psi_1, \psi_2, \ldots\}$ of nonnormed wave functions. The coordinate q and the momentum p are self-adjoint operators. The addition of the wave functions is always possible and we denote it with the sign $+$. The multiplication by the complex number is always possible also, and we state the set of complex numbers $C = \{c_0, c_1, c_2, \ldots\}$. Let $c_0 = i = \sqrt{-1}$. There are equalities in EQM, for example, the Schrödinger equation for an oscillator, $pp\psi_1 + c_1 qq\psi_1 = 0$. Of course, the number c_1 means the squared frequency of the oscillator. As another example, we may take the commutation relation $qp\psi_2 + c_0 c_0 pq\psi_2 = c_0\psi_2$.

Besides the described "dynamical part," EQM contains an "observer's part" also. Here the mean value of a self-adjoint operator A of an observable is postulated to be equal to the scalar product $(\psi^*, A\psi)$, where ψ is the wave function and the asterisk means the transition to the adjoint wave function. For simplicity, we may limit ourselves to A's that are polynomial in q and p.

In order to get the elementary quantum logic EQL we must first of all fix some wave function and declare it as the truth. Let it be the zero function $\psi = 0$. The symbols $\psi_0, \psi_1, \psi_2, \ldots$ are the proposition variables of EQL. The sign $+$ is the binary logical connective and $q, p, c_0, c_1, c_2, \ldots$ are the unary connectives of EQL. Formulas of EQL are constructed by means of the standard recursion and form the set F. So, EQL is a usual propositional logic with one binary logical connective and a countable set of unary connectives.

It is enough to consider the equalities of EQM with zero right-hand side only. Any such equality of EQM is considered as a formula of EQL, coincident with the left-hand side of the equality. So, the commutation relation of EQM turns into the formula $qp\psi_2 + c_0c_0pq\psi_2 + c_0c_0c_0\psi_2$ of EQL (note that $c_0c_0\psi_2 = -\psi_2$). All the correct EQM equalities are considered as theorems of EQL. Of course, the problem of a good axiomatization for EQL is outside our discussion.

We see that EQL accumulates the "dynamical part" of quantum mechanics in the first place. It is in contrast to the popular quantum logic discovered by Birkhoff and von Neumann, which accumulates the "observer's part."

The "observer's part" of EQM may be reflected in the theory of valuation of EQL. A valuation is a reflection $v: F \to L$, where L is the set of complex numbers.

We may get a valuation in the following way. First, any propositional variable $\psi \in \Psi$ is represented as a vector $\bar{\psi}$ of the complex Hilbert Space HS. Then $+$ is represented as the addition of vectors in HS, any logical connective $c \in C$ is represented as the multiplication of vectors by some complex number, and q and p are represented as some self-adjoint operators in HS. Then by the usual recursion, any logical formula $f \in F$ is represented as a vector $\bar{f} \in HS$. Now we define $v(f) = (\bar{\psi}_0^*, \bar{f})$. We call this valuation canonical if for any theorem t of EQL, $v(t) = 0$ and $\bar{\psi}_0$ is normed, that is, $(\bar{\psi}_0^*, \bar{\psi}_0) = 1$.

We believe that any $f \in F$ is a theorem of EQL if and only if $v(f) = 0$ for any canonical valuation. So, EQL is a complex-number-valued logic.

Note that $v(A\psi_0) = (\bar{\psi}_0^*, A\bar{\psi}_0)$. So, the mean values of observables are the truth-values in EQL. Analogously, the squared modulus $|v(f)|^2$ of the truth-values in EQL are the probabilities of the quantum transitions.

We have seen the equality of rights: resolving of his equalities (physicists) and proving of his theorems (logicists).

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